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A MANUAL
ON
EARTHWORK

GRAHAM

A MANUAL
ON
EARTHWORK.

BY

ALEX. J. S. GRAHAM, C.E.

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TO

R. P. BRERETON, ESQ.

CIVIL ENGINEER,

THE FOLLOWING

'MANUAL ON EARTHWORK'

IS

RESPECTFULLY DEDICATED,

AS

A TRIBUTE OF EARLY FRIENDSHIP AND ESTEEM,

BY

HIS OBLIGED SERVANT,

THE AUTHOR.

18 DUKE STREET, WESTMINSTER:
August 17, 1868.

PREFACE.

This Manual contains an investigation of all the principles necessary for the measurement and calculation of earthwork.

A general formula is given for curving, which renders the use of tables unnecessary. It may be easily retained in the memory for field practice. Several useful methods for treating curves are also given. The manner of setting out half-widths will be found simple and accurate, especially in sidelong ground. It affords all the data necessary for calculating contents, without the aid of cross sections, wherever the surface of the ground maintains a regular inclination; as may be seen by the tabular form proposed.

But the principal advantages of the following system are the correct estimation of excavations in sidelong ground, as to the cubical capacity, the acreage occupied, and the areas of the slopes; besides some expressions for the volume and surfaces of the wedge-shaped ends, whether their edges be square or oblique to the centre line; and for the several forms of trapezoidal solids that occur in practice.

The system of Tabulation proposed is less liable to error than that obtained from tabular numbers, as the figures are taken directly from the section, or half-width book.

For general or Parliamentary estimates, formulæ (*h*), (*l*), (*q*), and (*s*), as tabulated in examples 1, 4, 7, and 8, will be found, in most cases, sufficiently correct. But, for accuracy, tabulation from cross-sectional areas, as by formulæ (*l*), (*o*), and (*n*), and examples 3, 6, and 9, will be necessary, wherever the lateral inclination of the surface is considerable. Where the surface warps, or where the rate of inclination, transversely, differs within two chains' distance, the most accurate methods to be followed are those pointed out by formulæ (*h*), (*i*), (*n*), and examples 1, 2, and 5.

Where cross sections are necessary, the central and side heights can be obtained by scale from the line that equates the contour.

ALEX. J. S. GRAHAM,
Resident Engineer,
Forest of Dean Central Railway.

NIBLEY: *January* 1863.

A MANUAL

ON

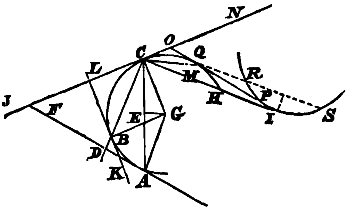
EARTHWORK.

ON CURVES.

The terminal points of curves are always equidistant from the intersection of the tangents to those points.

In marking out curves the angles FAC, AGC, DBA, and JFA (fig. 1), are generally used. The ∠ FAC is equal to half the ∠ AGC at the centre. The ∠ DBA is equal to the internal and opposite angles BAC and BCA, or the ∠ FAC subtended by the same arcs. Therefore the ∠ DBA is also equal to half the central angle AGC. The ∠ JFA is equal to the internal and opposite angles FAC and FCA, which are equal. Therefore the ∠ JFA is equal to twice FAC, or to the central ∠ AGC.

FIG. 1.



When the ∠ FAC cannot be had by observation, any ∠ FCD may be set off, and the ∠ FDC taken; then $\frac{\angle \text{FCD} + \angle \text{FDC}}{2} = \angle$ FAC, which may be set off at C or D, or any point along the tangent selected for the commencement of the curve.

If the radius of the arc ABC is required, the angle FAC = ∠ DAC is obtained; as also the length of the chord AC, or the latter may be calculated by measuring AD, and taking the angle ADC; for 180° − (∠ DAC + ∠ ADC) : AD :: ∠ ADC : AC. This saves the trouble of measuring the line AC.

Then as sin ∠ FAC = sin ∠ AGE, : half the chord = AE :: sin 90° : to the rad. : = AG.

If the point B is known to be in the curve, as in the renewal of curves previously staked, it is preferable to use the angle DBA in the above proportion. Or use the ∠ FAB and half the chord AB.

In some cases the irregularity of the ground may prevent observations of the point C, from either A, B, or D; then half the external angle JFA formed by the intersection of the tangents AF and CF, or half the central angle AGC formed by the intersection of the radii, may be used, or the line AG or GC may be measured.

Having found the radius, the angle of deflection per chain may be found as follows.

Let θ = the angle of deflection from the tangent for one chain's length of the curve, and r = the radius.

Then $2\theta : 1 :: 360 : 2r \times 3{\cdot}1416$; whence

$\frac{360}{4\theta} = 3{\cdot}1416r$ or $\frac{90}{\theta} = 3{\cdot}1416r$; whence

$$\theta = \frac{90}{3{\cdot}1416r} \text{ and } r = \frac{90}{3{\cdot}1416\theta} \text{ or}$$

$$\theta = \frac{28{\cdot}64789}{r} \text{ and } r = \frac{28{\cdot}64789}{\theta};$$

multiplying the numerators by 60, these give

$$\theta = \frac{1718{\cdot}8734}{r} \text{ and } r = \frac{1718{\cdot}8734}{\theta},$$

answering to minutes and decimals.

As derivable from these it may be stated, that if $\theta' =$ any angle at the centre, $\dfrac{\frac{\theta'}{2}}{\frac{28{\cdot}64789}{r}} = \dfrac{r\theta'}{57{\cdot}29578} =$ he length of the arc; and if $\theta'' =$ any angle at the cir-umference $\dfrac{r\theta''}{28{\cdot}64789} =$ the length of the arc.

If the length of the arc ABC is known, the angle deflection is equal to the angle FAC divided by it. he length ABC may not be had always accurately ough from the survey. However, if it be obtained m the survey, and if the curve, set out by the angle $\dfrac{\angle\text{FAC}}{\text{rc ABC}}$, does not terminate in C, it will come so e to it as to give the actual length of the arc ABC neasurement; and then the curve can be staked with the exact angular deflection without any er calculation.

the angle FAC is always equal to the angle DBA, line AB is set off at will, and the instrument *l along it*, at that angle, until the side DB, when produced, passes through the point C, B will be a point in the curve, and

$$\sin \angle \text{DAB} : \frac{\text{AB}}{2} :: \sin 90^\circ : \text{rad. AG.}$$

This appears to be the easiest method of finding the radius in long curves, and is sufficiently accurate when AB is not taken less than 10 chains.

In this instance, the point to commence the curve is B. To do this, an angle KBA = the angle DAB must be set off at B from the line BA. Then the instrument pointing in the line KL will be a tangent to the curve; from which the curve may be marked off backwards to A and forwards to C.

In this manner a tangent may be set off to any point in a curve; as it is only necessary to sight the chord of an arc backwards, and set off from it as many times the angle of deflection as there are chains in the arc. This is always necessary when the curvature changes from one radius to another without an intervening tangent; as, for instance, the angle of deflection per chain might be 1° from B to A, and 1° 30′ from B to C and the common tangent KL. The junction of curves and tangents is seldom at the end of a chain; therefore the deflection for the additional links is found by multiplying the angle for one chain by them. Thus, a curve of 1° deflection per chain gives ·30 degrees, or 18 minutes for 30 links.

If in curving along ABC it is required to find the point C, such that a tangent near that point shall be also a tangent to the curve RIS, it will be seen that the production of the chord CH, cutting at or near I, *shows the tangent* point M to be somewhere between

C and H. Produce the tangent CN, and from the point of intersection O, between it and the tangent PO of the curve RIS set off QO=OC. Measure the arc CQ, find the deflection the curve gives for that length, and if it agree with the angle OCQ, taken by observation, the curve terminates in Q. If it disagree, set off a tangent from Q, and if it cut inside the other curve at R and S, bisect RS, and set a pole at right angles to it at I. Take the angle between QS and IQ, continue the curve to M, a farther length due to half the angle SQI, and the tangent from thence will also be very nearly a tangent to the curve RIS. However it will still cut a little inside RIS, and it may be necessary to move the point M a few links farther on.

If a table of natural sines is not at hand, and the point B is fixed, we have, by putting AB$=a$ and DB, a perpendicular dropped from B upon AF, $=y$,

$$r=\frac{a^2}{2y} \text{ for the radius of a curve, and}$$

$$\frac{57{\cdot}29578y}{a^2}=\frac{3437{\cdot}747y}{a^2}=\theta \text{ in minutes and decimals}$$

of a minute.

Putting AD$=a'$ and DB$=y'$, we have

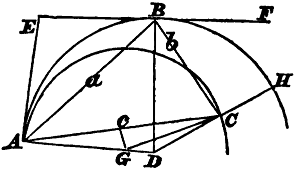
$$r=\frac{a'^2+y'^2}{2y'} \text{ for the radius of a curve, and}$$

$$\frac{57{\cdot}29578y'}{a'^2+y'^2}=\frac{3437{\cdot}747y'}{a'^2+y'^2}=\theta \text{ in minutes and decimals}$$

of a minute.

In deep rock cutting, or where the slopes are steep, and in rough ground, the operation of curving is sometimes entirely confined within the formation width, and no straight lines can be had. In this case a random curve ABH (fig. 2) may be run out with an assumed angle of deflection to any point B, from which the opposite end C of the required curve AC may be observed, and the $\angle$ FBC and the line BC obtained.

FIG. 2.



Thus: $\frac{28{\cdot}64789}{\theta}=r=\text{BD}$, and EF being tangent at B, $90° - \angle \text{ABE} = \angle \text{ABD}$, $180° - (\angle \text{ABE} + \angle \text{FBC}) = \angle \text{ABC}$, and $2 \angle \text{ABD} = \angle \text{ADB}$.

Putting $a=\text{AB}$, $b=\text{BC}$, and $c=\text{AC}$, we have

$$\sin \angle \text{ABD} : \frac{28{\cdot}6478}{\theta} :: \sin \angle \text{ADB} : a = \frac{28{\cdot}6478 \sin \angle \text{ADB}}{\theta \sin \angle \text{ABD}}$$

also putting $\angle \text{BAC} = \text{A}$, and $\angle \text{ACB} = \text{B}$, and $\angle \text{ABC} = \text{C}$, we have $180° - \angle \text{ABC} = \angle (\text{A} + \text{B})$, and

$$\frac{a-b}{a+b} \times \tan \frac{\text{A}+\text{B}}{2} = \tan \frac{\text{A}-\text{B}}{2}.$$

The angle answering to this tangent added to or subtracted from $\frac{\text{A}+\text{B}}{2}$, according as b is greater or less than a, gives the angle A, which added to the $\angle$ EAB gives $\angle$ EAC, the angle of the curve. To find r', we have

$$\sin \text{A} : b :: \sin \text{C} : c, \text{ and}$$

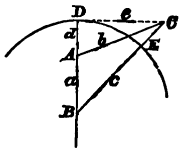
$$\sin \angle \text{EAC} = \angle \text{AGC} : \frac{c}{2} :: \sin 90° : \text{AG} = r'; \text{ then}$$

$\frac{28{\cdot}6478\dot{9}}{r'} = \theta'$ the true angle of deflection per chain.

This plan may be also used in open ground in some cases.

Where tables are not at hand, the angles of a triangle may be had as follows:—

FIG. 3.



Let BAC (fig. 3) be the given triangle, and the ∠ BAC the given angle.

Let BC$=c$, AB$=a$, AC$=b$, AD$=d$, DC$=e$.

Then $c^2=(a+d)^2+e^2=a^2+2ad+d^2+e^2$. But $b^2=d^2+e^2$,

$$\therefore\ c^2=a^2+b^2+2ad, \text{ and } \frac{c^2-(a^2+b^2)}{2a}=d.$$

Also as $b^2-d^2=e^2$, and the arc DE$=e^2-\frac{e^3}{3}+\frac{e^5}{5}-\frac{e^7}{7}$ &c. we have

$$6{\cdot}2832\ (a+d) : 360^\circ :: e-\frac{e^3}{3}+\frac{e^5}{5}-\frac{e^7}{7} \text{ \&c.} : \theta'$$

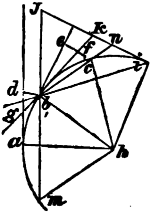
the number of degrees in the arc DE or the angle ABC. The angle ACB$=180^\circ-(\angle$ BAC$+\angle$ ABC).

Hence $\theta'=\frac{57{\cdot}29578}{a+d}\times\left(e-\frac{e^3}{3}+\frac{e^5}{5}+\frac{e^7}{7}, \text{ \&c.}\right)$ and $\frac{\theta'\ (a+d)}{57{\cdot}29578}=$ DE the length of the arc.

The sine of an angle BAC$=\sqrt{1-\left(\frac{a^2+b^2-c^2}{2ab}\right)^2}$

(See *Law's Tables*, page 160, formula 16.)

FIG. 4.



In laying out curves by the production of chords of equal length, as in fig. 4, we have—

$y = \frac{a^2}{r}$ where $y =$ the offset ec. $a =$ the chord bc, or ab, produced from b to e; and as the angles gba and fbc made by chords of equal arcs, ab and bc, with the tangent gf, are equal, and $gba =$ the vertical $\angle ebf$, the angle $fbc = \angle ebf$. Therefore $ef = fc = \frac{ec}{2} = \frac{y}{2}$, or the offset from the tangent is half that from the produced chord. Consequently the first offset $db = \frac{y}{2}$ or $\frac{a^2}{2r}$. In setting out on this principle, the chain must be laid along ab, and the point b found by moving it until d comes in line with ga. If the length be measured along ad there will be an error. Likewise, as the angle bec cannot be set out without a curving instrument made to suit the variation of that angle, the line be produced from a, through b, must be measured; a pin left at e, and the end of the chain moved from e to c until $ec = y$, the calculated offset.

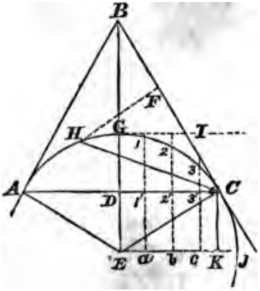
As the angle $fbc = \frac{\angle bhc}{2}$, or half the central angle $2 \angle fbc = \angle ebc = \angle bhc$, and the triangles ecb and chb are similar; $\therefore bh : bc :: bc : ce$, or $r : 1 :: 1 : y$, or $\frac{1^2}{r} = y$ or $\frac{792}{r} = y$ in inches, if r be in chains, as there *are 792 inches in a chain.*

ose $bc = 1$ chain, $bi = 2$ chains.

$r : 1 :: 1 : y = ec$, and $r : 2 :: 2 : y' = iJ$.

e $y' = 4y$, or the offset increases as the square chords. But the offsets from the tangent fc increase more rapidly. For $bc : bf :: bn : bk$, $> cn$, $\therefore bf > fk$, or $bk < 2bf$, $\therefore$ the square of ss than four times the square of bf, and as the is four times fc, it is evident that the offsets e tangent increase more than as the squares of ths measured on the tangents.

Fig. 5.



rinciple of ordinates tageously applied in ut small curves from 500 feet radius on ound. In fig. 5, the may be had by the previously stated, ines AD, DB, and BA d, then

BA :: AD : AE = , the radius;

the curve is less , let a perpendicular from H upon CB, the tangent to the other ex- of the curve CGH; then putting $CF = a'$, $FH = y'$,

$= r$, as previously stated.

from the equation of the circle, taking E as in, and $Ea = x$, $Eb = x'$, $Ec = x''$, $EK = x'''$, &c., , for the ordinates from the diameter EJ,

$-x^2 = a1$, $\sqrt{r^2 - x'^2} = b2$, $\sqrt{r^2 - x''^2} = c3$,

$$\sqrt{r^2-x'''^2}=\text{K}4,\ \&\text{c.},$$

and it is evident that these values deducted from the radius, or

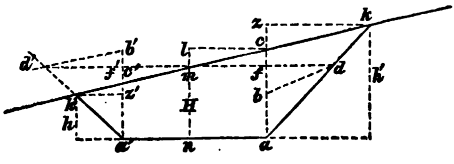
$$r-\sqrt{r^2-x^2},\ r-\sqrt{r^2-x'^2},\ r-\sqrt{r^2-x''^2},$$
$$r-\sqrt{r^2-x'''^2},$$

give the offsets from the tangent GI at 1, 2, 3, &c., or that the value of KC deducted from those ordinates will give 1 1′, 2 2′, 3 3′, &c., the points 1′, 2′, 3′, corresponding with *a*, *b*, *c*, being taken at any distances apart that are suitable to the surface of the ground. The versed sine $\text{DG}=r-\text{KC}=r-\sqrt{r^2-x'''^2}$.

ON HALF-WIDTHS.

As sidelong ground has, in most cases, a regular inclination, the following system for finding the half-widths and the values of *h* and *h′* (fig. 6) is humbly suggested for saving the trouble of making and plotting cross sections.

Fig. 6.



Let kk' represent the natural surface of the ground, H the central height of cutting, $lc=mf=na=\frac{w}{2}$ half the formation width of a cutting, and $d=cf$ the difference of level between l and c; also $fd=rH$ and $zk=rh'$.

Then, drawing bd parallel to ck, we have

$$ab : bd :: ac : ck. \quad \text{But } bd : ck :: rH : zk$$

$$\therefore ab : rH :: ac : zk \text{ and } \frac{rH \times ac}{ab}=zk.$$

$$\text{But } \frac{w}{2} : d :: rH : fb=\frac{2drH}{w} \therefore ab=H-\frac{2rdH}{w}.$$

Substituting this value in the preceding formula, we have

$$rh' \text{ or } zk=\frac{rH \times ac}{H-\frac{2rdH}{w}}=\frac{wr \times ac}{w-2rd},$$

and putting x for ac, we have $\frac{wrx}{w-2rd}=zk.$

$$\text{Also } rH : H :: zk : za=\frac{H \times zk}{rH}=\frac{zk}{r} \text{ or}$$

$$za=\frac{wx}{w-2rd}=h'.$$

In like manner h may be found, and the half-widths will be $\frac{w}{2}+rh'$ and $\frac{w}{2}+rh$ respectively.

On the lower side of the cutting, if a', b', c', &c.,

$w = 30$ $\quad r = 1\frac{1}{2}$

Length in Chains	Half-width $= \frac{w}{2} + rk$	$k = \frac{wry}{w - 2rd'}$	$y = \text{H}$ + Diff.	Diff. of Level	Left	H and central Readings	Right	Diff. of Level	$x = \text{H}$ + Diff.	$k' = \frac{wrx}{w - 2rd}$	Half-width $= \frac{w}{2} + rk'$
1·00	28·03	8·69	7·88	·94	5·44	6·94 4·50	3·44	1·06	8·00	8·95	28·42
2·00	32·70	11·80	9·28	2·14	6·15	7·14 4·01	1·15	2·86	10·00	14·00	36·00
3·00	38·08	15·39	12·18	2·09	6·09	10·09 4·00	·09	3·91	14·00	22·98	49·47

represent corresponding points to *a*, *b*, *c*, &c., on the upper, we shall find, by putting $a'c'=y$, and $c'f'=d'$

$$\frac{wry}{w-2rd'}=zk \text{ and } \frac{wy}{w-2rd'}=za=h.$$

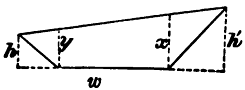
By inverting the figure it will be seen that these formulæ are also applicable in embankments. The accompanying form for the half-width book will suit for taking the field notes. H, in the centre column, is the height from the formation level to the surface of the ground given by the longitudinal section. A line is drawn between it and the reading of the centre stake to which it refers, and horizontally in line with this reading are those of x and y, taken on the right and left, at a distance from the centre equal to one-half the formation width. The centre, right, and left readings are therefore all the observations necessary in the field, as the other columns can be filled indoors, and the half-width stakes placed subsequently, according to the distances shown under that head.

INVESTIGATION OF THE FORMULÆ FOR CALCULATING EARTHWORK.

In the case of sidelong ground, the values of x and y in the following investigations may not be always had from observation, but may be calculated thus in *fig.* 7:—

$w+r(h'+h) : h'-h :: rh' : h'-x$, whence

Fig. 7.



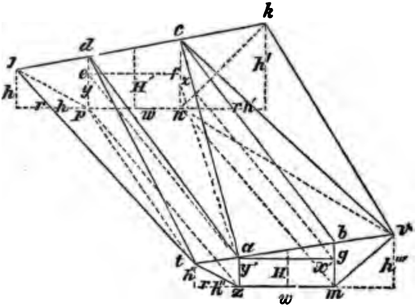
$$(a)\ x=\frac{h'(w+2rh)}{w+r(h'+h)},$$ a

$$(b)\ y=\frac{h(w+2rh')}{w+r(h'+h)},$$

TRAPEZOIDAL FORMULA.

Let the adjoining fig. 8 represent a trapezo
solid contained between two parallel cross section

Fig. 8.



any cutting. Suppose it to be cut by a plane a
parallel to *pnmz*. Also by two vertical planes a
and *bmnc*; and by another vertical plane *afc* al
the diagonal *af*. Then if $pd=y$, $nc=x$, $az=y'$,
$mb=x'$, $L=pz$, and $w=zm$

$$\frac{(x'-y')+(x-y')}{2} \cdot \frac{Lw}{3} = gbcfa$$

$$\frac{(y-y')+(x-y')}{2} \cdot \frac{\text{L}w}{3} = cfdea$$

$$y'w\text{L} = azmgfnep$$

$$\therefore \frac{\text{L}w}{3} \cdot \frac{(x'+y+2x)-4y'}{2} + y'w\text{L}$$

$$=(x'+y+2x-4y'+6y')\frac{\text{L}w}{6} =$$

$$(\text{A}) = \left\{ x'+y+2\,(x+y') \right\} \frac{\text{L}w}{6} = azmbcndp$$

e content of the figure, when the surface slopes both ays, from *cb* to *ad* and from *dc* to *ba*, exclusive of e truncated pyramids at the sides formed by the opes.

To find the truncated pyramids *tazjdp* and *bvmcnk*, uppose a plane passing through *tpd*. Then if *r* xpress the ratio of the slopes,

$$\frac{rhy}{2} \cdot \frac{\text{L}}{3} = \text{pyramid}\, jdpt.$$

$$\frac{\text{L}\,(y+y')}{2} \cdot \frac{rh''}{3} = rh''\,(y+y')\,\frac{\text{L}}{6} = \text{pyramid}\, pdazt.$$

Hence,

$$\left[rhy+rh''\,(y+y') \right\} \frac{\text{L}}{6} = r\,\left\{ hy+h''\,(y+y') \right\} \frac{\text{L}}{6} =$$

e truncated pyramid *jdptaz*. Similarly

$$r \left\{ h'x + h'''\,(x+x') \right\} \frac{\text{L}}{6} = kcnvbm.$$

ombining these with A we have

$$(\text{B})\ \left(w \left\{ x'+y+2\,(x+y') \right\} + r\,\left\{ hy+h'x+h''\,(y+y')+h'''\,(x+x') \right\} \right) \frac{\text{L}}{6}$$

=the content of the whole figure *jkvtzmnp*, when the surface slopes both ways from *kv* to *jt*, and from *jk* to *vt*.

Putting H′ and H for the central heights at the ends, when the surface is level on the cross section, and slopes from *jk* to *tv* only,

$$\left.\begin{array}{ll} h,\ h',\ x,\ y, & \text{each} = H' \\ h'',\ h''',\ x',\ y', & \text{each} = H \end{array}\right\} \text{consequently (B) becomes}$$

$$\Big(w\ \Big\{H + H' + 2\,(H' + H)\Big\}$$

$$+\ r\ \Big\{H'^2 + H'^2 + H\ (H' + H) + H\ (H' + H)\Big\}\Big)\frac{L}{6} =$$

$$\Big\{3w\ (H' + H) + r\ \ (2H'^2 + 2H'H + 2H^2)\Big\}\ \frac{L}{6} =$$

$$\text{(C)}\ \Big\{3w\,(H' + H) + 2r\,(H'^2 + H\,H + H^2)\Big\}\ \frac{L}{6} =$$

the content of the trapezoidal solid.

(B) may be also written—

$$(\alpha)\ \Big(w\ \Big\{x' + y + 2\,(x + y')\Big\}$$

$$+\ r\left\{\frac{hy^2 - h''y'^2}{y - y'} + \frac{h'x^2 - h'''x'^2}{x - x'}\right\}\Big)\frac{L}{6}$$

or,

$$(\beta)\ \Big(w\ \Big\{x' + y + 2\ (x + y')\Big\}$$

$$+\ r\Big\{hy + h''y' + \sqrt{hy \times h''y'} + h'x + h'''x' + \sqrt{h'x\ \times\ h'''x'}\Big\}\Big)\frac{L}{6}$$

or,

$$(\gamma)\ \Big(w\ \Big\{x'+y+2\,(x+y')\Big\}$$

$$+r\left\{\frac{h+h''}{y+y'}\cdot(y^2+yy'+y'^2)+\frac{h'+h'''}{x+x'}\cdot(x^2+xx'+x'^2)\right\}\Big)\frac{\text{L}}{6}$$

In the latter $\frac{h+h''}{y+y'}$ and $\frac{h'+h'''}{x+x'}$ are the ratios for correcting the truncated pyramids when the surface is warped. x, x', y, and y', can be had by (a) and (b) or observation on the ground.

On examining (A) we find that $\frac{x'+y}{2}\cdot w$ is the area due to the average height between m and p, and $\frac{x+y'}{2}\cdot w$, that to the average height between z and n. Therefore, this formula expresses twice the middle area on mp, and four times the middle area on zn, when $x+y'>x'+y$, and *vice versâ* when $x'+y>x+y'$. Consequently this formula is to be written

$$\frac{\text{L}w}{6}\Big\{x+y'+2\,(x'+y)\Big\}\text{ when } x'+y>x+y'.$$

When the surface is much warped, twice the middle area on mp will not be equal to the sum of the areas on zm and pn, nor will four times the area on zn be equal to four times the middle area on ww; consequently, where tabulation by cross sections is adopted, as in formula (D), the sections are to be taken nearer than one chain apart when the surface of the ground is much warped, or the rate of inclination on the cross sections differs within a length of two chains.

(C) may be written—

$$\left\{3w\,(H'+H)+r\,(2H'^2+2H'H+2H^2)\right\}\frac{L}{6}=$$

$$\left(3w\,\{H'+H\}+r\left\{H'^2+(H^2+2H'H+H^2)+H^2\right\}\right)\frac{L}{6}=$$

$$\left(3w\,\{H'+H\}+r\left\{H'^2+(H'+H)^2+H^2\right\}\right)\frac{L}{6}=$$ th

content.

Here we have

$2w\,(H'+H)+r\,(H'+H)^2=$ four times the middle are

$w\,(H'+H)+r\,(H'^2+H^2)=$ the sum of the end areas

for $\frac{H'+H}{2}=$ the middle height, and

$$w\,\frac{H'+H}{2}+r\,.\,\frac{H'+H}{2}\times\frac{H'+H}{2}=w\,\frac{H'+H}{2}+r\,.\,\frac{(H'+H}{4}$$

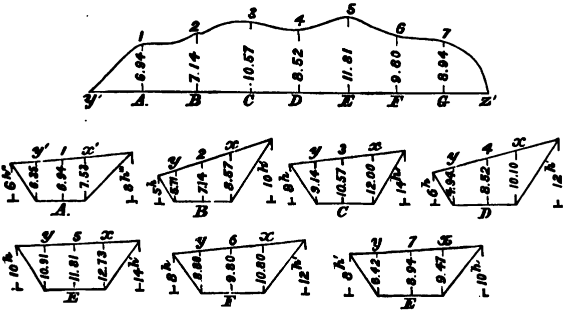
= the middle area, and four times the middle area = $2w\,(H'+H)+r\,(H'+H)^2$.

Hence the practical rule for finding the volume c a trapezoidal solid of this description. 'To the su of the areas of each end add four times the middl area, and multiply by one-sixth of the length.'

In adopting (C) as the principle for tabulating b cross sections, it is well to remark again that whereve the cross-sectional inclination differs much within tw chains' length, it is necessary for extreme accuracy t take the observations at half chains, or nearer, as ma be seen by considering the remarks upon (A). Othe *wise it* will be necessary to tabulate by (*m*), (*p*), an (*u*), *as in Examples* 1, 2, and 5.

According to this principle, dividing the cutting, fig. 9, into an even number of parts, one chain in length each, = L, and taking the solids 2 chains long, =2L, we shall have

FIG. 9.



$$\left\{2\,(C+E,\ \&c.)+4\,(B+D+F,\ \&c.)+A+G\right\}\frac{2L}{6}=$$

$$^{*}(D)\left\{2\,(C+E,\ \&c.)+4(B+D+F,\ \&c.)+A+G\right\}\frac{L}{3}=$$

the volume of the cutting, exclusive of the end wedges A*y′* and G*z*.

A and G represent the terminal cross sections A1 and G7; C, E, &c., those at the odd points C3, E5, &c.; B, D, F, &c., those at the even points B2, D4, F6, &c.

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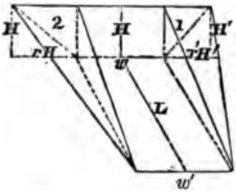
* See Hughes' Tables, published by Effingham & Wilson. 1846.

WEDGES AND TRAPEZOIDAL SOLIDS.

When the back is a regular figure and the edge parallel to it (fig. 10), we have

FIG. 10.



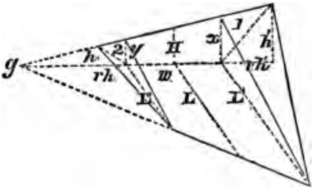
$\frac{w\text{LH}}{2}$ = content of wedge on H

$\frac{r\text{LH}}{3}$ = content of the pyramids on 1 and 2. Hence

$$\frac{w\text{LH}}{2}+\frac{r\text{LH}}{3}=\text{H}(3w+2r\text{H})\,\frac{\text{L}}{6}\ (\text{E})$$

gives the volume of the whole figure; where L = the length, H = the height of the back, r = the ratio of the slopes, and w = the formation width.

FIG. 11.



In the case of sidelong ground forming oblique wedges (fig. 11) we have

$$\text{H}=\frac{x+y}{2}\ \text{and}$$

$$\text{H} : \text{L} :: x : \frac{\text{L}x}{\text{H}}=\text{L}'(c)$$

Likewise $\text{H} : \text{L} :: y : \frac{\text{L}y}{\text{H}}=\text{L}''\ (d)$. Then

$$\left(\frac{x+h'}{2}\,.\,rh'-\frac{rh'^2}{2}\right)\frac{\text{L}'}{3}=\frac{rh'x}{2}\,.\,\frac{\text{L}}{3}=$$

content of pyramid on 1.

$$\left(\frac{y+h}{2}\,.\,rh-\frac{rh^2}{2}\right)\frac{\text{L}''}{3}=\frac{rhy}{2}\,.\,\frac{\text{L}'}{3}=$$

content of pyramid on 2.

By substituting for L' and L'' their equivalents in (*c*) and (*d*), the sum of these

$$=\frac{r\text{L}}{6\text{H}}(h'x^2+hy^2)\ (e).$$

For that portion of the wedge on H we have

$$x-y : w :: x : \frac{wx}{x-y}=xg, \text{ and}$$

$$x-y : w :: y : \frac{wy}{x-y}=yg,$$

$$\therefore \left(\frac{\text{L}'x}{2}\cdot\frac{wx}{3(x-y)}\right)-\left(\frac{\text{L}''y}{2}\cdot\frac{wy}{3(x-y)}\right)$$

$$=\frac{\text{L}'x^2-\text{L}''y^2}{x-y}\cdot\frac{w}{6}=\text{the content.}$$

By substituting for L' and L'' their equivalents in (*c*) and (*d*) this becomes

$$\frac{\text{L}x^3-\text{L}y^3}{\text{H}(x-y)}\cdot\frac{w}{6}=\frac{\text{L}(x^3-y^3)}{\text{H}(x-y)}\cdot\frac{w}{6}=\frac{w\text{L}}{6\text{H}}(x^2+y^2+xy)\ (f)$$

Combining (*e*) and (*f*) we have

$$\frac{r\text{L}}{6\text{H}}(h'x^2+hy^2)+\frac{w\text{L}}{6\text{H}}(x^2+y^2+xy)=$$

$$\frac{\text{L}}{6\text{H}}\left\{r(h'x^2+hy^2)+w(x^2+y^2+xy)\right\}=(g)$$

the whole volume of the trapezoidal wedge, (fig. 11) By substituting for *x* and *y* their values in (*a*) and (*b*) we have, in terms of the side heights h and h',

$$\frac{\mathrm{L}}{6\mathrm{H}}\left\{\frac{rh'\left\{h'(w+2rh)\right\}^2+rh\left\{h(w+2rh')\right\}^2+w\left\{h'(w+2rh)\right\}^2+w\left\{h(w+2rh')\right\}^2+w\left\{h'(w+2rh)\times h(w+2rh')\right\}}{\left\{w+r(h'+h)\right\}^2}\right\}=$$

$$=\frac{\mathrm{L}}{6\mathrm{H}}\left\{\frac{(rh'+w)\,.\left\{h'(w+2rh)\right\}^2+(rh+w)\,.\left\{h(w+2rh')\right\}^2+w\left\{h'(w+2rh)\times h(w+2rh')\right\}}{\left\{w+r(h'+h)\right\}^2}\right\}=$$

$$\frac{\mathrm{L}}{6\times 27\mathrm{H}}\left\{\begin{array}{l}\left.\begin{array}{r}(rh+w)\,h^2\times 1+\\+(rh'+w)\,h'^2\times 1-\end{array}\right\}r\,.\,\dfrac{2w\,(h'-h)+r\left\{2(h'^2-h^2)-(h'-h)\right\}}{\left\{w+r\,(h'+h)\right\}^2}\\[2ex]+\,whh'\times 1-\dfrac{r^2\,(h'-h)^2}{\left\{w+r\,(h'+h)\right\}^2}\end{array}\right. \qquad (\mathrm{F})$$

by various reductions and dividing by 27 for cubic yards. When $h=h'=\mathrm{H}$, this becomes

$$\mathrm{H}\,(3w+2r\mathrm{H})\,\frac{\mathrm{L}}{6\times 27}\text{ the same as }(\mathrm{E}).$$

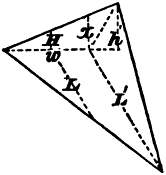
When $h=o$ it becomes

$$\frac{\mathrm{L}}{6\times 27\mathrm{H}}\cdot\frac{h'^2w^2}{w+rh'}\text{ the same as }(\mathrm{G}).$$

When the terminal wedge is a pyramid (fig. 12) we have $\frac{wh'}{2} \cdot \frac{\text{L}'}{3}$ = the content. This, by substituting for x and L$'$ their values in (a) and (c)

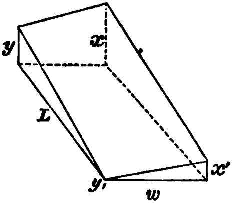
$$\frac{w}{2} \cdot \frac{\text{L}xh'}{3\text{H}} = \frac{\text{L}w}{6\text{H}} \cdot \frac{h'^2\,(w+2rh)}{w+r\,(h'+h)}.$$

FIG. 12.



But as $h=o$, it takes the form of $\frac{\text{L}}{6\text{H}} \cdot \frac{h'^2w^2}{w+rh'}=$(G), the volume of the pyramidal wedge (fig. 12). The three preceding forms of wedges are those that occur most frequently in the formation of earthworks, and have therefore been transformed to expressions containing h and h' instead of x and y, as the latter may not be had, after a cutting or embankment is finished, without separate calculations.

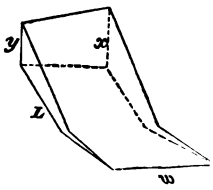
FIG. 13.



From (A) when $y'=o$

(H) $w\,(x'+y+2x)\frac{\text{L}}{6}=$ content of fig. 13.

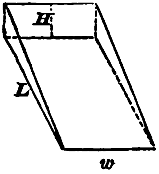
FIG. 14.



When x' and $y'=o$

(I) $w\,(2x+y)\frac{\text{L}}{6}=$ content of fig. 14.

FIG. 15.



When x' and $y' = o$, and $x + y = \text{H}$,

$$(\text{J})\ \frac{\text{HL}w}{2} =$$

content of fig. 15.

In (H) when there are side slopes, we have from (B)

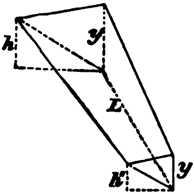
$$\left\{w\,(x' + y + 2x) + r\,(hy + h'x + h''y) + h'''\,(x + x')\right\}\frac{\text{L}}{6}\ (\text{K}).$$

$$\text{In (I)}\ \left\{w\,(2x + y) + r\,(h'x + hy)\right\}\frac{\text{L}}{6}$$

In (J) $\text{H}\,(3w + 2r\text{H})\,\frac{\text{L}}{6}$, which is the same as (E).

For a truncated pyramid (fig. 16) we have from the second term of (B)

FIG. 16.



$$r\left\{hy + h''(y + y')\right\}\frac{\text{L}}{6}\ (\text{L}),\ \text{or from}$$

$$(\alpha)\ r\,.\,\frac{hy^2 - h''y'^2}{y - y'}\,.\,\frac{\text{L}}{6},\ (\text{M}),\ \text{or}$$

$$(\text{N})\ r\,.\,(hy + h''y' + \sqrt{hy \times h''y'})\,\frac{\text{L}}{6}$$

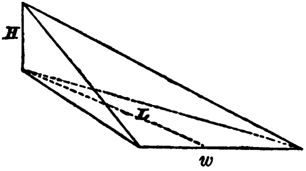
from (β), or

$$(\text{O})\ r\,\frac{h + h''}{y + y'}\,(y^2 + yy' + y'^2)\,\frac{\text{L}}{6}\ \text{from}\ (\gamma).$$

When the ground is level on the cross section $h = y$ and $h'' = y'$, $\therefore$ we have

(P) $r(y^2+yy'+y'^2)\,\frac{L}{6}=$ the content of fig. 16.

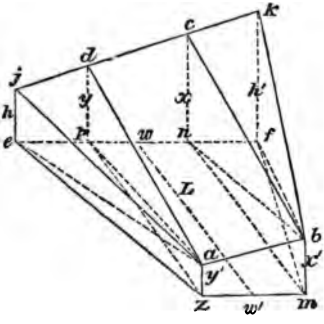
FIG. 17.



For a wedge like fig. 17 we have, by the first formula for (A),

$$\frac{wL}{2}\cdot\frac{H}{3}=wH\cdot\frac{L}{6}\ (Q).$$

FIG. 18.



When the formation-width is greater at one end than the other, or $w > w'$ (fig. 18), we have from (Q)

$$\frac{w-w'}{2}\cdot\frac{L}{2}\cdot\frac{y'}{3}=$$

$$\frac{y'(w-w')}{2}\cdot\frac{L}{6}=azep,$$

and

$$\frac{x'(w-w')}{2}\cdot\frac{L}{6}=bmfn.$$

Also $\frac{h+y}{2}\cdot\frac{w-w'}{2}\cdot\frac{L}{3}=\frac{(h+y)\,.\,(w-w')}{2}\cdot\frac{L}{6}=aepdj,$

and $\frac{(h'+x)\,.\,(w-w')}{2}\,.\,\frac{L}{6}=bfncd.$ Hence

$$\left(\frac{y'(w-w')}{2}+\frac{x'(w-w')}{2}+\frac{(h+y)\,.\,(w-w')}{2}+\frac{(h'+x)\,.\,(w-w')}{2}\right)\frac{L}{6}=$$

$$(\text{R})\ (y'+x'+y+x+h+h')\,\frac{w-w'}{2}\,.\,\frac{L}{6}=$$

the sum of the contents of the side wedges, in which

$$(\text{S})\ (y'+y+h)\,\frac{w-w'}{2}\,.\,\frac{L}{6}=azejdp, \text{ and}$$

$$(\text{T})\ (x'+x+h')\,\frac{w-w'}{2}\,.\,\frac{L}{6}=bmfnck.$$

As from (A) we get the content of *azmbcnpd*, if we add (A) and (R) together we have

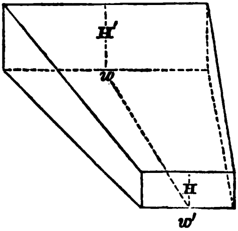
$$(\text{U})\ \left\{w'\{x'+y+2\,(x+y')\}\right.$$
$$\left.+\frac{w-w'}{2}\,(y'+x'+y+x+h+h')\ \right\}\frac{L}{6}=$$

the cubical volume of the whole figure *azmbkfej*, or $\frac{w}{2}\left\{\,2\,(y+y')+h+h'\,\right\}\frac{L}{6}$ when $w'=o$, as in this case $x'=y'$ and $x=y$.

In (U) when $w=w'$, the second term vanishes, and we have (A).

In (S) and (T) when the wedges are not similar, *ep* and *nf* are to be used instead of $\frac{w-w'}{2}$.

FIG. 19.



In fig. 19, where the surface is level on the cross section, and slopes only in one direction, from w to w', we have in (U)

h, y, x, h', each $= H'$
y' and x' each $= H$,

$$\therefore \left\{3w'(H'+H)+(w-w')\,.\,(2H'+H)\right\}\frac{L}{6}=$$

$$(V)=\left\{w'(H'+2H)+w(H+2H')\right\}\frac{L}{6}=$$

the cubic volume, or

$$w\,(H+2H')\,\frac{L}{6} \text{ when } w'=o.$$

From (U) when $y'=o$, fig. 20,

$$\left\{w'(x'+y+2x)+\frac{w-w'}{2}(x'+y+x+h+h')\right\}\frac{L}{6}=(W)$$

FIG. 20.

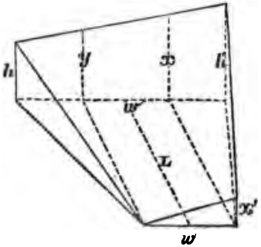
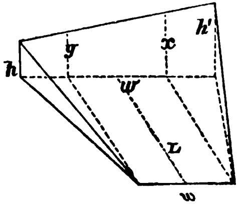


FIG. 21.



the cubic content, or

$$\frac{w}{2}(x'+2x+h+h')\frac{\text{L}}{6} \text{ when } w'=o;$$

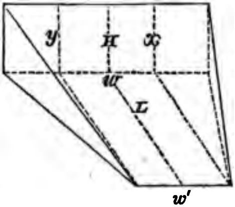
and when x' and $y'=o$, fig. 21,

$$\left\{w'(y+2x)+\frac{w-w'}{2}(y+x+h+h')\right\}\frac{\text{L}}{6}=(\text{X})$$

the volume, or

$$\frac{w}{2}(y+x+h+h')\frac{\text{L}}{6} \text{ when } w'=o.$$

FIG. 22.



and when x' and $y'=o$, and x and $y=\text{H}$, fig. 22,

$$\text{H}(w'+2w)\frac{\text{L}}{6}=(\text{Y})$$

the cubic content, or

$$2\text{H}w\frac{\text{L}}{6} \text{ when } w'=o.$$

In a truncated pyramid of the form shown by fig. 23, we have from (M)

$$(\text{Z})\ \left\{r\cdot\frac{y^2(h+h')-y'^2(h''+h''')}{y-y'}\right\}\frac{\text{L}}{6}=$$

the content; or from (L)

$$\left(r\{y(h+h')+(y+y')\cdot(h''+h''')\}\right)\frac{\text{L}}{6}$$

or from (O)

$$r\,\frac{h+h'+h''+h'''}{y+y'}(y^2+yy'+y'^2)\frac{\text{L}}{6}$$

And when the surface is level on the cross sections, as h and $h'=y$, and h'' and $h'''=y'$, we have (fig. 24) from (Z)

FIG. 23.

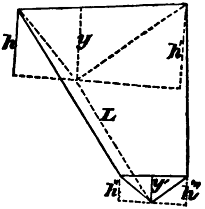
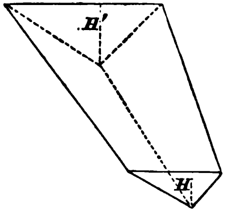


FIG. 24.



$$\left(2r\ .\ \frac{H'^3-H^3}{H'-H}\right)\frac{L}{6}=\text{the content; or}$$

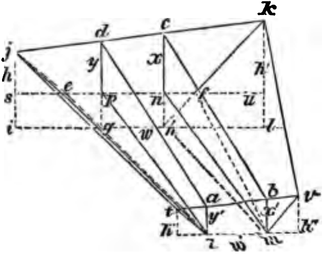
$$(A_1)\ \ 2r\,(H^2+H'H+H'^2)\,\frac{L}{6}\ \text{the same as twice (P).}$$

(Q), when there are side slopes, becomes

$$H\left\{w+r\,(h'+h)\right\}\frac{L}{6}\ (B_2).$$

Fig. 25 represents fig. 18 with side-slopes. Its cubic capacity may be found by adding the side figures *jdpeatz* and *kcnfbvm* to the central portion *azmbcnpd*, as given by formula (A).

Fig. 25.



Let *je* and *kf* be produced until they meet *dp* and *cn* produced in *g* and *n'*, and join *gz* and *n'm*.

Then as $r \times pg = ep = \frac{w-w'}{2} \therefore pg = \frac{w-w'}{2r} = si = ul.$

Add $\frac{w-w'}{2r}$ to *js* and we have *ji*, which call H. Add it to *ku* and we have *kl*, which call H$'$. Also call *dg*, Y, and *cn'*, X.

Now from the second term of (B)

$$r\left\{\mathrm{HY}+\mathrm{H'X}+h''(\mathrm{Y}+y')+h'''(\mathrm{X}+x')\right\}\frac{\mathrm{L}}{6}=$$

the content of the truncated pyramids *jdgatz* and *kcn'bvm*. Also

$$\left(\frac{w-w'}{2}\cdot\frac{w-w'}{2r}\right)\frac{\mathrm{L}}{6}=$$

the pyramid *epgz*, and

$$2\left(\frac{w-w'}{2}\cdot\frac{w-w'}{2r}\right)\frac{\mathrm{L}}{6}=\frac{(w-w')^2}{2r}\cdot\frac{\mathrm{L}}{6}=$$

the content of both pyramids, *epgz* and *fnn'm*. Therefore

$$(\mathrm{C}_1)\quad\left(r\left\{\mathrm{HY}+\mathrm{H'X}+h''(\mathrm{Y}+y')+h'''(\mathrm{X}+x')\right\}-\frac{(w-w')^2}{2r}\right)\frac{\mathrm{L}}{6}=$$

the content of *jdpeatz* and *kcnfbvm*. On combining this with (A) we have

$$(\mathrm{D}_1)\quad\left(w'\left\{x'+y+2(x+y')\right\}+r\left\{\mathrm{HY}+\mathrm{H'X}+h''(\mathrm{Y}+y')+h'''(\mathrm{X}+x')\right\}-\frac{(w-w')^2}{2r}\right)\frac{\mathrm{L}}{6}=$$

the whole cubical volume *jkfetvmz*. When $w'=o$ this becomes, as $X=Y$ and $x'=y'$,

$$\left(\left\{Y(H+H')+(Y+y')\,.\,(h''+h''')\right\}-\frac{w^2}{2r}\right)\frac{L}{6}$$

When there are side slopes and the surface is level on the cross section (see fig. 19), we have from (D_1)

$$\left\{3w'(y+y')+2r(H^2+Hy'+y'^2)-\frac{(w-w')^2}{2r}\right\}\frac{L}{6}=(E_1)$$

$$\text{and }\left\{2r(H^2+Hy'+y'^2)-\frac{w^2}{2r}\right\}\frac{L}{6}\text{ when }w'=o.$$

When $y'=o$ (see fig. 20) we have from (D_1)

$$\left\{w'(x'+y+2x)+r\{HY+H'X+h'''(X+x')\}\right.$$
$$\left.-\frac{(w-w')^2}{2r}\right\}\frac{L}{6}=(F_1)$$

$$=\left(r\{HY+H'X+h'''(X+x')-\frac{w^2}{2r}\right)\frac{L}{6}\text{ when }w'=o.$$

When x' and $y'=o$ we have likewise (see fig. 21)

$$\left\{w'(2x+y)+r(HY+H'X)-\frac{(w-w')^2}{2r}\right\}\frac{L}{6}=(G_1)$$

$$=\left\{r(HY+H'X)-\frac{w^2}{2r}\right\}\frac{L}{6}\text{ when }w'=o;\text{ and}$$

when x' and $y'=o$, $H'=X=Y=H$, and $x=y$ (see fig. 22), we have

$$\left(3w'y+2rH^2-\frac{(w-w')^2}{2r}\right)\frac{L}{6}=(H_1)\text{ where }y=H,\text{ in}$$

fig. 22.

$$=\left(2rH^2-\frac{w^2}{2r}\right)\frac{L}{6}\text{ when }w'=o.$$

APPLICATION OF THE PRECEDING FORMULÆ.

As sometimes cuttings and embankments are formed with retaining walls, and without slopes, of the form shown by *azmbcnpd* (fig. 8); the formula (A) gives in such cases

$$w\Big(\cdot 4074\,\{x' + y + 2(x + y') + 3\mathrm{s}\}\Big)$$

for the volume of a cutting or embankment in cubic yards; where s = the sum of the intermediate heights on the lower and upper side; x' and y', the terminal heights at the beginning; x and y those at the end, and the constant $\cdot 4074 = \frac{66}{6 \times 27}$.

For if we suppose a similar figure to extend from *dpnc* (fig. 8) on the one side, and another from *azmb* on the other, we shall find that x' will be taken once for fig. 8, and twice for the adjacent figure; also y' twice for fig. 8, and once for the adjacent figure. Therefore x' and y' will be taken three times each, so likewise will x and y, for any number of intermediate sections. At the first section x' will be taken once, y' twice. At the last section x will be taken twice, and y once, we have therefore

$$w\Big(\cdot 4074\,\{\,x' + y + 2(x + y') + 3\mathrm{s}\}\Big) =$$

$$(h) = \cdot 2037\; w\{2\,(x' + y) + 4\,(x + y') + 6\;\mathrm{s}\}$$

for the content of such a cutting, exclusive of the end *wedges.*

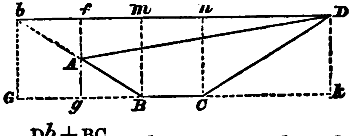
When the ground is much warped, and there are side slopes, the quantities can be had most accurately by combining the latter equation (h) with the second term of (B). We shall then have

$$(i)\ \cdot 2037\,w\left\{6\mathrm{S}+2\,(x'+y)+4\,(x+y')\right\}$$
$$+\cdot 4074\,r\,(s+s'+s''+s''')+y'+z'$$

for the content of a cutting; where x' and y', x and y are as in fig. 8, supposing several sections to intervene between them; $\mathrm{S}=$ the sums of x and y at all the intermediate sections, $s=$ the sums of $h''\,(y'+y)$, $s'=$ the sums of $h'''\,(x'+x)$ at all but the last section, $s''=$ the sums of hy, and $s'''=$ the sums $h'\,x$ at all but the first section, and $y'+z'$, the contents of the end wedges, $\mathrm{A}y'+\mathrm{G}z'$ (fig. 9) to be found by formulæ (E), (F), (G), &c.

In order to apply formula (D)* we have to find a general expression for the cross-sectional areas A1, B2, &c., as shown by ABCD (fig. 26).

FIG. 26.



Here $\dfrac{\mathrm{D}b+\mathrm{BC}}{2}\,.\,\mathrm{D}k=$ area $\mathrm{BCD}b$, and

$\dfrac{\mathrm{D}b}{2}\,.\,\mathrm{A}f=$ area $\mathrm{AD}b$.

---

[*] This formula (D), which is the application of the Prismoidal, holds good where the ground is not much warped, and the cross sections are taken closely; also in all cases where the heights are taken from the longitudinal section.

$$\therefore\ (1)\ \frac{\mathrm{D}k\,(\mathrm{D}b+\mathrm{BC})-\mathrm{D}b\times\mathrm{A}f}{2}=\text{area ABCD.}$$

Put $\mathrm{D}k=h'$, $\mathrm{A}g=h$, $\mathrm{BC}=w$, and $r=$ the ratio of the slopes; then $n\mathrm{D}=rh'$, also $mb=rh'$, and $mn=w$; $\therefore\ \mathrm{D}b=2rh'+w$, and $\mathrm{D}b+\mathrm{B}c=2rh'+2w$. Also $\mathrm{D}b\times\mathrm{A}f=(2rh'+w)\times(h'-h)$. Hence from (1)

$$\frac{h'(2rh'+2w)-(2rh'+w)\,.\,(h'-h)}{2}=$$

$$\frac{h'w+2rhh'+hw}{2}=\frac{w}{2}(h'+h)+rhh' \quad (2)$$

gives the area ABCD.

The multiplier in the general formula (D) becomes $\frac{\mathrm{L}}{3\times27}$ for cubic yards, or ·8148 when $\mathrm{L}=66$ feet, and 1·2345 when $\mathrm{L}=100$ feet. Consequently, putting y' and z' for the content of the end wedges we have

$$(j)\ \cdot8148\left\{2\,(\mathrm{C}+\mathrm{E},\ \&\text{c.})+4\,(\mathrm{B}+\mathrm{D}+\mathrm{F},\ \&\text{c.})+\mathrm{A}+\mathrm{G}\right\}$$

$+y'+z'=$ the cubical volume of the whole cutting from y' to z' in fig. 9. When the distances are 100 feet apart this becomes

$$(k)\ 1\cdot2345\left\{2\,(\mathrm{C}+\mathrm{E},\&\text{c.})+4(\mathrm{B}+\mathrm{D}+\mathrm{F},\ \&\text{c.})+(\mathrm{A}+\mathrm{G})\right\}+y'+z'$$

In (2) if the sums of $(h'+h)=\mathrm{S}$, and those of their products $h\times h'=s$ for the cross-sections at C, E, &c., then $w\mathrm{S}+2rs$ gives twice the sum of their areas, or $2\,(\mathrm{C}+\mathrm{E},\ \&\text{c.})$ in (j) and (k). Likewise in the cross-sections at B, D, F, &c., if the sums of $(h'+h)=\mathrm{S}'$, and those of $h\times h'=s'$, $2w\mathrm{S}'+4rs'$ gives four times the *sum of their areas*, or $4\,(\mathrm{B}+\mathrm{D}+\mathrm{F},\ \&\text{c.})$ Finally, if the sums of $(h'+h)=\mathrm{s}''$, and those of $h\times h'=s''$, in the cross-sections at A and G, $\frac{w}{2}\,\mathrm{s}''+rs''$ gives the sum of the areas at A and G. Hence (j) may be written

$$\cdot8148\left\{(w\mathrm{s}+2rs)+(2w\mathrm{s}'+4rs')+(\frac{w}{2}\,\mathrm{s}''+rs'')\right\}+y'+z'$$

and (k)

$$1\cdot2345\left\{(w\mathrm{s}+2rs)+(2w\mathrm{s}'+4rs')+(\frac{w}{2}\,\mathrm{s}''+rs'')\right\}+y'+z'$$

or

$$(l)\quad \cdot2037w\,(4\mathrm{s}+8\mathrm{s}'+2\mathrm{s}'')+\cdot4074r\,(4s+8s'+2s'')+y'+z'$$

and

$$(m)\quad \cdot3086w\,(4\mathrm{s}+8\mathrm{s}'+2\mathrm{s}'')+\cdot6172r\,(4s+8s'+2s'')+y'+z'$$

The values of $\cdot2037w$, $\cdot4074r$, &c., are given in the Table of Ratios.

When $\mathrm{L}=33$, 50, 16·50 or 25 feet, one half or one fourth the tabular values of these constants is to be used.

Also when tabulation from the heights on the longitudinal section is necessary, the values of $2h$ are used instead of those of $(h'+h)$, and those of h^2 instead of $h\times h'$, as may be seen on comparing examples 3 and 4.

Should V-shaped drains, in sidelong ground (fig. 23) be calculated in successive lengths, the tabulation of (Z) as derived from (O) is the best.

If h'' and h''', h'''' and h''''' represent the side heights, y' and y'' the middle heights, at the terminal sections. Also s, the sums of $h+h'$; s', the sums of y; s, the sums of y^2; and s', the sums of $y'y$ at all the intermediate sections. Then

$$\left\{ r\,\frac{2s+h''+h'''+h''''+h'''''}{2s'+y'+y''}\,.\,2s+s'+y'^2+y''^2 \right\}\frac{\mathrm{L}}{6\times 27}=$$

$$(n)\quad \cdot 4074r\left(\frac{2s+h''+h'''+h''''+h'''''}{2s'+y'+y''}\,.\,2s+s'+y'^2+y''^2\right)\sim$$

the content, which added to the end pyramids gives the cubic volume of such a drain. The end pyramids equal the areas of the terminal sections multiplied by one third of their respective lengths. If the values of y are not taken by observation, they may be had from $y=\frac{2h'h}{h'+h}$ (3).

An easier tabulation for V-drains may be had by cross-sectional areas, unless the surface of the ground be much warped.

The area of a V-shaped drain (figs. 23 and 24) is

$$\left\{ r\,(h'+h)\,.\,\frac{h'+h}{2} \right\}-\left(\frac{rh'^2}{2}+\frac{rh^2}{2}\right)=$$

$$r\,\frac{(h'+h)^2-(h'^2+h^2)}{2}=rh'h\ (4);$$

consequently if $s=h'\times h$ at C, E, &c., $s'=h'\times h$ at B, D, F, &c., $s''=h'\times h$ at A and G (fig. 9), we shall have from (l)

$$(o)\quad \cdot 4074r\,(4s+8s'+2s'')+y'+z'$$

for the cubical content when 66 feet lengths are used; and from (m)

$$(p)\quad \cdot 6172r\,(4s+8s'+2s'')+y'+z'$$

when the lengths are 100 feet.

Should the surface of the ground be level (fig. 24), *we have from* (A_1)

$$2r\,(2\text{S}+\text{S}'+\text{H}^2+\text{H}'^2)\,\frac{\text{L}}{6} =$$

$$(q)\ \cdot 4074r\left\{4\text{S}+2\,(\text{S}'+\text{H}^2+\text{H}'^2)\right\}$$

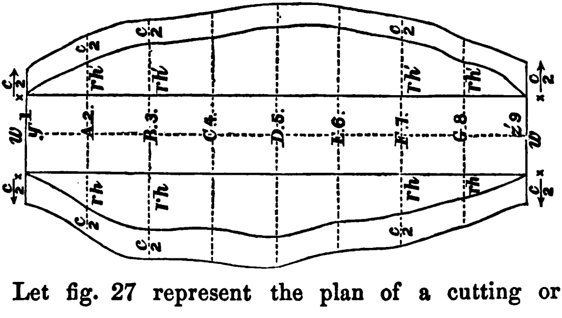
the content in cubic yards, exclusive of the end wedges y' and z', which may be found as in (n) from (4).

Here $\text{S}=$the sums of H'^2; $\text{S}'=$ the sums of $\text{H}'\text{H}$; H^2 and H'^2, the squares of the terminal heights.*

CALCULATIONS FOR LAND.

The quantity of land to be occupied by a cutting or embankment may be found as follows:—

FIG. 27.



Let fig. 27 represent the plan of a cutting or embankment where $\frac{c}{2}$ is the distance between the top of the slopes and the fence, w the formation width, and rh, rh', &c., the bases of the slopes. Dividing it into an *even* number of *parts*, and calling the ordinates at right angles to the centre line at each length y', A, B, C, &c., we have

---

* Supposing fig. 24 divided by several parallel sections, when H' at the end of the first length becomes H at the beginning of the second length, and H' at the end of the second length becomes H at the beginning of the third length, and so on.

$$(r)\ \frac{\text{L}}{3}\left\{4(\text{A}+\text{C}+\text{E}+\text{G})+2(\text{B}+\text{D}+\text{F})+y'+z'\right\}=$$

the area expressed in feet.

Putting S for the sums of $(h'+h)$ in A, C, E, &c., and S$'$ for those of $(h'+h)$ in B, D, F, &c., and $(w+c)=y'=z'$, we have

$$(s)\ \frac{\text{L}}{3}\left\{r(4\text{S}+2\text{S}')+3(n+1)\,.\,(w+c)\right\}\ \text{or}$$

$$\frac{\text{L}}{3}\left\{r(4\text{S}+2\text{S}')\right\}$$

when $(w+c)=o$, as in V-shaped drains, for the area of the land occupied; where $n=$ the number of *even* parts into which the cutting has been divided. (See Hughes's Tables, note 2.)

When these calculations are carried on continuously from cutting to embankment this formula holds good, whether the ends are square to the centre line or not. But when the land required for a single cutting (fig. 9) or embankment with oblique ends is to be computed, we have (fig. 11), by substituting for L$'$ and L$''$ their values in (c) and (d), and for x and y their values in (a) and (b),

$$\left.\begin{array}{l}\frac{Lx}{2H}\times rh' = \Delta \text{ on } 1.\\[1ex] \frac{Ly}{2H}\times rh = \Delta \text{ on } 2.\\[1ex] Lw \qquad = \square \text{ on } 3.\end{array}\right\} = (t)\ L\,(r\,.\,\frac{xh'+yh}{2H}+w) =$$

$$L\left(r\,.\,\frac{h'^2\,(w+2rh)+h^2\,(w+2rh')}{2H\,\{w+r\,(h'+h)\}}+w\right)$$

for the area of each oblique end, and for the land occupied by the remainder of the cutting

$$\frac{L}{3}\left\{4\,(B+D+F,\ \&c.)+2\,(C+E,\ \&c.)+A+G\right\}$$

or, putting S'' for the sum of $(h'+h)$ in A and G,

$$(u)\ \frac{L}{3}\left\{r\,(4S'+2S+S'')+3\,(n-1)\,.\,(w+c)\right\}=$$

the area, exclusive of the oblique ends, and when $w+c=o$, as in V-drains,

$$\frac{L}{3}\left\{r\,(4S'+2S+S'')\right\}$$

In (s) and (u) $\frac{L}{3}=\cdot07958$, and in (t) $L=\cdot2424$, for perches, when the distances are 66 feet long. If 100 feet long, these become ·1224 and ·3673 respectively.

AREAS OF SLOPES.

As $r\,(h'+h)$ expresses the bases of the slopes in fig. 27, sec. $\theta\,(h'+h)$ will express the lengths of the sloped surface on each side at A, B, C, &c., θ being the angle of the slope with the vertical. Consequently (t) becomes $\text{L}\left(\sec.\,\theta\,\frac{xh'+yh}{2\text{H}}\right)=$

$$(v)\qquad \text{L}\left(\sec.\theta\,.\,\frac{h'^2\,(w+2rh)+h^2\,(w+2rh')}{2\text{H}\,\{w+r\,(h'+h)\}}\right)$$

for the area in square feet of the sloped surface of the terminal triangles, and (u) becomes

$$\frac{\text{L}}{3}\left\{\sec.\,\theta\,(4\text{s}'+2\text{s}+\text{s}'')\right\}(w)$$

for the area of the intermediate slopes from A to G.

But if these calculations are carried on from cutting to embankment, we have from (s)

$$\frac{\text{L}}{3}\left\{\sec.\,\theta\,(4\text{s}+2\text{s}')\right\}(x)$$

for the superficies. The values of sec. θ, or $\sqrt{1+r^2}$ are given in the Table.

When $\text{L}=66$ feet, $\frac{\text{L}}{9}=7{\cdot}3333$ becomes the multiplier in (v) for square yards, and $\frac{\text{L}}{3\times9}=2{\cdot}4444$ in (w) and (x). If $\text{L}=100$ feet, these multipliers become 3·7037 and 11·1111 respectively.

TABLE OF RATIOS.

w	r	·2037 w	·4074 r	·3086 w	·6173 r	Sec. θ $\sqrt{1+r^2}$
15		3·0555		4·6297		
16	¼	3·2592	·1018	4·9384	·1543	1·0307
17		3·4629		5·2470		
18	½	3·6666	·2037	5·5557	·3036	1·1180
19		3·8703		5·8643		
20	1	4·0740	·4074	6·1730	·6172	1·4100
21		4·2777		6·4816		
22	1½	4·4814	·6111	6·7903	·9258	1·8027
23		4·6851		7·0989		
24	2	4·8888	·8148	7·4076	1·2345	2·2360
25		5·0925		7·7162		
26	2½	5·2962	1·0185	8·0249	1·5431	2·6925
27		5·4999		8·3335		
28	3	5·7036	1·2222	8·6422	1·8517	3·1622
29		5·9073		8·9508		
30	3½	6·1110	1·4259	9·2595	2·1603	3·6405
31		6·3147		9·5681		
32	4	6·5184	1·6296	9·8768	2·4690	4·1231
33		6·7221		10·1854		
34	5	6·9258	2·0370	10·4941	3·0862	5·0990
35		7·1295		10·8027		
36	6	7·3332	2·4444	11·1096	3·2035	6·0828

EXAMPLES.

Ex. 1.—Required the quantity of cutting in Fig. 9, supposing the formation width to be 30 feet, the values of h to be the side heights, and no slopes.

By (h)

S	
h	h'
5	10
8	14
6	12
10	14
8	12
37	62
	37

$99 \times 6 = 594$

92

$\cdot 2037 w = 6 \cdot 111 \times 686 = 4192 \cdot 14$

$\text{A}y' = 258 \cdot 37$

$\text{G}z' = 331 \cdot 34$

$4781 \cdot 85$ c.yds.

$$* \; 2(x' + x) = 2(8 + 10) = 36$$

$$4(y' + y) = 4(6 + 8) = \underline{56}$$

$$92$$

Wedges by $(f) = \frac{\text{WL}}{6\text{H}}(x^2 + y^2 + xy)$

$$\text{A}y' = \cdot 4074 \times 30 \, \frac{64 + 36 + 48}{7} = 258 \cdot 37 \text{ c.yds.}$$

$$\text{G}z' = \cdot 4074 \times 30 \, \frac{100 + 64 + 80}{9} = 331 \cdot 34 \quad ,,$$

* Note: —In the length A,B, $y' + x > x' + y$, therefore x' is taken twice only. In the length FG, $x' + y > x + y'$ therefore x is only taken twice; and for the same reasons y' and y are taken four times (see remarks upon formula A).

Ex. 2.—Required the same with sides sloped at $1\frac{1}{2}$ to 1.

By Formula (i)

S		s	s'	s''	s'''
y	x	$h'(y'+y)$	$h'''(x'+x)$	hy	$h'x$
5·71	8·57	73·80	128·80	28·55	85·70
9·14	12·00	74·25	205·70	73·12	168·00
6·94	10·10	128·64	309·40	41·64	121·20
10·91	12·73	107·10	273·96	109·10	178·22
8·80	10·80	197·10	329·42	70·40	129·60
		137·76	243·24	67·36	94·70
41·50 54·20	54·20	718·65	1490·52	390·17	777·42

$\overline{6 \times 95{\cdot}70} = 574{\cdot}20$

$34{\cdot}00 = 2(x'+x)$

$59{\cdot}08 = 4(y'+y)$

$\cdot 2037w = \overline{6{\cdot}111 \times 667{\cdot}28} = 4077{\cdot}74$

2063·47

$Ay' =$ 315·83

$Gz' =$ 428·21

6885·25 c.yds.

390·17
$718{\cdot}65 = s$
$1490{\cdot}52 = s'$
$777{\cdot}32 = s'''$

$\overline{3376{\cdot}66} \times \cdot 6111 = 2063{\cdot}47$

$2(x'+x) = 2(7{\cdot}53+9{\cdot}47) = 34{\cdot}00$

$4(y'+y) = 4(6{\cdot}35+8{\cdot}42) = 59{\cdot}08$

The end wedges are, by formula (F)

$$Ay' = \cdot 4074 \times \frac{1}{6{\cdot}94} \left\{ \begin{array}{l} (9+30) \times 36 \times 1{\cdot}1142 = 1564{\cdot}33 \\ (12+30) \times 64 \times \cdot 8858 = 2381{\cdot}03 \\ 30 \times 6 \times 8 \times \cdot 9965 = 1434{\cdot}96 \end{array} \right\} = \cdot 4074 \frac{5580{\cdot}32}{6{\cdot}94}$$

$= 315{\cdot}83$ c.yds.

$$Gz' = \cdot 4074 \times \frac{1}{8{\cdot}94} \left\{ \begin{array}{l} (12+30) \times 64 \times 1{\cdot}1025 = 2963{\cdot}52 \\ (15+30) \times 10 \times \cdot 8975 = 4038{\cdot}75 \\ 30 \times 8 \times 10 \times \cdot 9977 = 2394{\cdot}48 \end{array} \right\} = \cdot 4074 \frac{9396{\cdot}75}{8{\cdot}94}$$

$= 428{\cdot}21$ c.yds.

Ex. 3.—Required the same from cross-sectional areas, by formula (*l*).

	S	s		S′	s′		S″	s″
	$h'+h$	$h\times h'$		$h+h$	$h\times h'$		$h'+h$	$h\times h$
C	14 8	112	B	10 5	50	A	8 6	48
E	4 10	140	D	12 6	72	G	10 8	80
			F	12 8	96			
	46	252		53	216		32	128
	4	4		8	8		2	2
	$4S=184$	1008		424	$1744=8s'$		64	256
	$8S'=424$				$1008=4s$			
	$2S''=\ 64$				$256=2s''$			

$6{\cdot}111 \times 672 = 4106{\cdot}59$ $\qquad 3008 \times {\cdot}6111 = 1838{\cdot}18$

$$\begin{array}{r} 1838{\cdot}18 \\ Ay' = 315{\cdot}83 \\ Gz' = 428{\cdot}21 \\ \hline 6688{\cdot}81 \end{array}$$

Ay′ and Gz′: Wedges the same as in previous Example.

6688·81 c.yds.

Note. The difference between the results of this and the preceding Example exemplifies the error of the Prismoidal formula when the surface of the ground is warped.

Ex. 4.—Required the same, using the heights on the longitudinal section and formula (*l*).

	S	s		S′	s′		S″	s″
	$2h$	h^2		$2h$	h^2		$2h$	h^2
			B	14·28	50·97			
C	21·14	111·72				A	13·88	48·16
			D	17·04	72·59			
E	23·62	139·47				G	17·88	79·92
			F	19·60	96·04			
	44·76	251·19		50·92	219·60		31·76	128·08
	4	4		8	8		2	2
	4s=179·04	1004·76		407·36	1756·80=8s′		63·52	256·16

8s′=407·36   1004·76=4s

63·52   256·16=2s′

6·111×649·92=3971·66   3017·72 × ·6111=1844·13

1844·13

Ay′ = 311·22

Gz′ = 425·47

6552·48 c. yds.

The end Wedges by (E) are

Ay′=·4074 × 6·94 (3 × 30 + 2 × 1·5 × 6·94)=311·22 c. yds.

Gz′=·4074 × 8·94 (3 × 30 × 2 × 1·5 × 8·94)=425·47 c. yds.

Note. The differences between this and the two preceding Examples exemplify the error that results by using the heights on the Longitudinal Section instead of those obtained by Cross-sections, or Half-widths.

Ex. 5.—Required the quantity of cutting in Fig. 9, from the side heights h, h', &c., when $w=0$, or the cutting is V-shaped. Here by (3) the values of y, y', &c. $=\frac{2h'h}{h'+h}$ as they are not given by observation. Let $r=1\frac{1}{2}$ to 1, as before.

By formula (n)

S		S′	s	s′
h	h'	y	y^2	$y'y$
5	10	6·66	44·35	45·62
8	14	10·18	103·63	67·79
6	12	8·00	64·00	81·44
10	14	11·66	135·95	93·28
8	12	9·60	92·16	111·93
				85·25
37	62	46·10	440·09	485·31
62		2	2	880·18 = 2s
	2 × 99 = 198	92·20	880·18	46·92 = y'^2
$h''+h'''+h''''+h'''''$ =	32	15·73 = $y'+y''$		78·85 = y''^2
	230	107·93		

1491·26 × 1·3023 = 1942·07
58·66 = Ay'
97·77 = Gz'
2098·50 c. yds.

$$\frac{2s+h''+h'''+h''''+h'''''}{2s'+y'+y''}=\frac{230}{107·93}$$
= 2·1311
·6111 = ·4074r.
1·3023

End Pyramids by (4)

Ay' $= rh'h \cdot \frac{L}{3 \times 27}$ = 1·5 × 6 × 8 × ·8148
= 58·66 c. yds.

Gz' = „ „ = 1·5 × 8 × 10 × ·8148
= 97·77 c. yds.

Ex. 6.— Required the same from cross sectional areas.

By formula (o)

	$\frac{S}{h\times h'}$		$\frac{S'}{h\times h'}$		$\frac{S''}{h\times h'}$
		B	50		
C	112			A	48
		D	72		
E	140			G	80
		F	96		
	252		218		128
	4		8		2
	1008		1744		256

$$\begin{array}{r}
1008\\
1744\\
256\\
\hline
3008
\end{array} \times \cdot 6111 = \left.\begin{array}{r} 1838\cdot 18 \\ Ay' = \quad 58\cdot 66 \\ Gz' = \quad 97\cdot 77 \end{array}\right\} \text{Pyramids, as in Example 5.}$$

$$1994\cdot 61 \text{ c. yds.}$$

Here, again, the result derived from the Prismoidal formula is less than that from Trapezoidal.

Note. ...e Trapezoidal

Ex. 7.—Required the same when the surface of the ground is level on the cross-sections. Here the values of H will be the same as those of y in Example 5.

By formula (q).

S	S′
44·35	45·62
103·63	67·79
64·00	81·44
135·95	93·28
92·16	111·93
	85 25
440·09	485·31

4   46·92 $= \text{H}^2$

1760·36   78·85 $= \text{H}'^2$

$2 \times 611{\cdot}08 = 1222{\cdot}16$

1760·36

$\cdot 6111 \times 2982{\cdot}52 = 1822{\cdot}61$

$\text{A}y'$ 57·34

$\text{G}z'$ 96·36

1976·31 c. yds.

End pyramids by (4).

$$\text{A}y' = r\text{H}^2 \cdot \frac{\text{L}}{3 \times 27} = 1{\cdot}5 \times 46{\cdot}92 \times \cdot 8148 = 57{\cdot}34 \text{ c. yds.}$$

$$\text{G}z' = \text{,,} \qquad \text{,,} \qquad = 1{\cdot}5 \times 78{\cdot}85 \times \cdot 8148 = 96{\cdot}36 \text{ c. yds.}$$

Ex. 8.—Required the quantity of land occupied by a cutting when the ends are square to the centre line, and central heights are given. See figs. 9 and 27.

By formula (s).

	S		S′
	$2h$		$2h$
A	13·88	B	14·28
C	21·14	D	17·04
E	23·62	F	19·60
G	17·88		
	76·52		50·92
	4		2
	306·08		$101·84 = 2s'$

$4s = 306·08$
$2s' = 101·84$
$407·92 \times 1·5 = 611·88$

$n = 8,\ w = 30,\ c = 15.$

$$3(n+1).(w+c) = 3(8+1).(30+15) = 1215$$

$$\frac{611·88}{1215}$$

$$·07958 \times 1826·88 = 145·3831 \text{ perches}$$

or, $0a.\ 3r.\ 25·38p.$

Ex. 9.—Required the same when the ends are oblique, and the side heights given by cross-sections are used.

By formula (n).

	S		S′		S″
	$h'+h$		$h'+h$		$h'+h$
		B	10 5		
C	14 8			A	8 6
		D	12 6		
E	14 10			G	10 8
		F	12 8		
	46		53		32 = S

$$\begin{array}{r} 46 \\ 2 \\ \hline 2S = 92 \end{array} \qquad \begin{array}{r} 53 \\ 4 \\ \hline 212 = 4S' \end{array}$$

$2S = 92$
$4S' = 212$
$S'' = 32$

Oblique ends by formula (t).

$$\text{A to } y' = \cdot2424\left(r\,.\,\frac{h^2(w+2rh)+h^2(w+2rh')}{2H\{w+r(h'+h)\}}+w\right) =$$

Perches

$$= \cdot2424\left(1\cdot5\,\frac{64\,(30+18)+36\,(30+24)}{2\times6\cdot94\,\{30+1\cdot5\,(8+6)\}}+30\right) = \quad 9\cdot8484$$

$$\text{G to } z' = \cdot2424\left(1\cdot5\,\frac{100\,(30+24)+64\,(30+30)}{2\times8\cdot94\,\{30+1\cdot5\,(10+8)\}}+30\right) = \quad 10\cdot5647$$

$$3\,(n-1)\,.\,(w+c) = 3\,(8-1)\,.\,(30+15) = 945$$

$$\begin{array}{r} 504 \\ \hline \cdot07958\times1449 = \quad 115\cdot3114 \end{array}$$

Total perches 135·7245

or 0a. 3r. 15·72p.

Ex. 10.—Required the square yards of slopes corresponding with the last example. Here the slopes being $1\frac{1}{2}$ to 1, sec. $\theta = 1{\cdot}8027$, and by formula (w)

$$2{\cdot}4444 \times 1{\cdot}8027 \times 336 \qquad \overset{\text{Sqr. Yds.}}{=1480{\cdot}5906} \text{ from A to G,}$$

and by (v) $$7{\cdot}3333\left(1{\cdot}8027 \,.\, \frac{64\,(30+18)+36\,(30+24)}{2\times 6{\cdot}94\,\{30+1{\cdot}5\,(8+6)\}}\right) = \;\; 93{\cdot}6733 \;\; \text{,, A to } y'\text{, and}$$

$$7{\cdot}333\left(1{\cdot}8027 \,.\, \frac{100\,(30+24)+64\,(30+30)}{2\times 8{\cdot}94\,\{30+1{\cdot}5\,(10+8)\}}\right) \qquad = \underline{119{\cdot}8525} \;\; \text{,, G to } z'.$$

Total 1694·1164 square yards.

CONCLUDING OBSERVATIONS.

After examining the methods proposed in the previous pages, for obtaining the data and calculations necessary for earthwork operations, considerable advantages in facility and correctness will be discovered.

In the system for Half-widths, three observations with the level are all that are necessary for laying off the side stakes—instead of five, and sometimes seven—and the results arrived at are precise; whereas the ordinary method is, at best, only continual approximation. Where the surface of the ground is uniform, it also affords all the *data* supplied by a regular system of cross sections, and, therefore, dispenses with all the labour and time required in making, plotting, and calculating the areas of those sections. It also diminishes, to an important extent, the liabilities to error consequent upon these operations; as, with the exception of the three central columns of the table, obtained by observation, the others are filled up from the results of calculations subsequently arrived at indoors and at leisure.

When regular cross sections are made, the formula for obtaining the areas saves much trouble, and is far more accurate than the subdivision of those sections into triangles, finding the area of each separately, and taking their sum for the whole area. In such cases, the heights h and h' can be read off a scale from a line *equating the* surface.

The Prismoidal formula, which is that hitherto *applied for finding* the cubical capacity of earthworks, *is here shown to* be a particular case of the gener formula (B), which has been termed the Trapezoidal formula; not only as it distinguishes it from the former, but as it expresses more clearly the varied forms which earthwork solids present in practice. The prismoidal formula gives inaccurate results when applied to ground where the parallel sections are not of an uniform inclination—that is, where the surface is warped; as may be seen by the remarks upon formula (A), and a comparison of tabulations Nos. 2 and 3. However, where central heights from the longitudinal section are only used, it is correct; and it is hoped that the method of application here given affords a much simpler method of tabulation than any from tables.

The formula for oblique and truncated wedges have not been hitherto investigated for practical use, and will prove very convenient to the resident or assistant-engineer, or foreman of works, in progress measurements; as during the execution of works these forms continually occur; and as the expressions are reduced to the ruling principle of measurement by heights—one length and breadth only being requisite—it is evident that these formula will much expedite the field operations.

The formulæ for V-drains, although not much required hitherto, are likely to be so, in consequence of the system of drainage coming into action; also as they are derivable directly from those for ordinary earthworks, they are necessary for the completion of the system. The formulæ for obtaining the areas of slopes are given also for the same purpose, as well as *for obtaining* more accurate data for general or *parliamentary* estimates.

The investigations of the preliminary formulæ give some curious results, that may be found useful in the higher branches of calculation, and in the cubature of solids generally. They provide for every form of wedge and truncated solids likely to be ordinarily met with in field-work practice; and the ease with which they may be combined—none going higher than simple equations, and all symmetrically derived—renders it likely that they supply accurate results for every possible irregularity of shape in more complex figures.

It was not thought necessary to give separate diagrams for all the variations of these formulæ, as they are derivable from the primary forms, and may be easily sketched with the pencil, if necessary in practical operations.

The observations on curving are derived from several years' investigation and practice; and as such, may be depended upon for immediate application. A detailed investigation of the theories on this subject has been so frequently given, and would so much exceed our intended limits, that it was considered best simply to state some of the most useful results.

Example No. 1 shows the method of tabulation for cuttings without side slopes; and although these are not of frequent occurrence, except in the neighbourhood of large cities, where retaining walls are built; yet it may be sometimes applicable, and it exemplifies more clearly than the others the principle of diagonal dimensions, as shown by formula (A).

Example No. 2 shows the most correct method of *tabulating for* earthwork quantities, as proved by *formula (B)*. It is more operose than that derived *from the Prismoidal* formula, as shown in exampl

No. 3 ; but as the surface of the ground is warped in most cases, and the latter does not then give accurate results, it is preferable in all cases of final measurements, or those made for letting works to contractors.

Example No. 4 is that by which parliamentary estimates can be most rapidly obtained; and as, in such cases, the heights can only be had from the longitudinal section, it affords as much accuracy as the conditions admit—being, as before remarked, a particular form of the trapezoidal formula, and true in its results when the surface, on the cross section, has an uniform inclination; as by hypothesis it has wherever these heights only are given.

These remarks apply likewise to Examples 5, 6 and 7, which are applications of the same principles to v-shaped drains.

Examples 8 and 9 give the areas of land occupied by earthworks, without waiting for the completion of the surveys. In parliamentary calculations the former may be continued to any extent, from cutting to embankment consecutively; as, in such cases, the ends, whether square or oblique to the centre line, do not affect the results. The latter is applicable to isolated cuttings or embankments where greater accuracy is required; and, consequently, the side heights and the obliquity of the ends are taken into account.

Example No. 10 shows the method of obtaining the number of square yards of slopes to be trimmed and soiled in the completion of earthworks; and as these operations include no inconsiderable item in calculations of this description, it will doubtless prove acceptable.

These tabulations, therefore, include every conditio that is likely to occur in earthwork practice, and the facilities presented by Examples 4, 7, 8, and 10, for arriving, without the use of tabular numbers, at correct estimates for parliamentary or general purposes, as well as the superior accuracy of the whole system in theory and tabulation, will, it is hoped, obtain for this little work a favourable consideration and preference from civil engineers and other professional men who may be interested in the subject.

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